

Bulk Viscous cosmological models in Lyra geometry

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Abstract

We have investigated an LRS Bianchi Type I models with bulk viscosity in the cosmological theory based on Lyra's geometry. A new class of exact solutions have been obtained by considering a time-dependent displacement field for a constant value of the deceleration parameter and viscosity coefficient of bulk viscous fluid is assumed to be a power function of mass density. The physical behaviour of the models is also discussed.

Keywords : Cosmology; L R S Bianchi type-I models; Lyra geometry; Bulk viscous universe

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1 Introduction

After Einstein, in 1917, developed his general theory of relativity, in which gravitation is described in terms of geometry, Weyl, in 1918, proposed a more general theory in which electromagnetism is also described geometrically. However, this theory, based on non-integrability of length transfer, had some unsatisfactory features and did not gain general acceptance. Later Lyra [1] suggested a modification of Riemannian geometry, which may also be considered as a modification of Weyl's geometry, by introducing a gauge function into the structureless manifold which removes the non-integrability condition of the length of a vector under parallel transport and a cosmological constant is naturally introduced from the geometry. In subsequent investigations, Sen [2] & Sen and Dunn [3] proposed a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry.

Halford[4] pointed out that the constant displacement vector field ϕ_i in Lyra's geometry plays the role of a cosmological constant in the normal general relativistic treatment. Halford[5] showed that the scalar-tensor treatment based on Lyra's geometry predicts the same effects, within observational limits, as in Einstein's theory. Several authors (Bhamra[6], Karade and Borikar[7], Kalyanshetti and Wagmode[8], Reddy and Innaiah[9], Beesham[10], Reddy and

Venkateswarlu[11] and Soleng[12]) have studied cosmological models based on Lyra's geometry with a constant displacement field vector. However, this restriction of the displacement field to be a constant is a coincidence and there is no *a priori* reason for it. Singh and Singh[13]–[15] and Singh and Desikan[16] have studied Bianchi Type I, III, Kantowski-Sachs and a new class of models with a time dependent displacement field and have made a comparative study of Robertson-Walker models with a constant deceleration parameter in Einstein's theory with a cosmological term and in the cosmological theory based on Lyra's geometry. Recently Pradhan and Vishwakarma[17] investigated a new class of an LRS Bianchi Type-I cosmological models in Lyra geometry. Though the displacement vector has no clear and unambiguous interpretations some efforts have been made to treat the constant displacement vector as the analogue of the cosmological constant. Soleng[12] has pointed out that the cosmologies based on Lyra's manifold with a constant gauge vector ϕ will either include a creation field and be equal to Hoyle's creation field cosmology[18]–[20] or contain a special vacuum field which together with the gauge vector may be considered as a cosmological term. In the latter case, the solutions are same as those of general relativistic cosmologies with a cosmological term. Recently Behnke *et al.*[21] also pointed out to an alternative description of the new cosmological supernova data without a Λ -term as evidence for Weyle's geometry of similarity, where Einstein's theory takes the form of the conformal-invariant theory of a massless scalar field[22] –[25]. As it has been shown by Weyl already in 1918, conformal-invariant theories correspond to the relative standard of measurement of a conformal-invariant ratio of two intervals, given in the geometry of similarity[26] as a manifold of Riemannian geometries connected by conformal transformations. This ratio depends on nine components of metrics whereas the tenth component became the scalar dilation field that cannot be removed by the choice of the gauge. In the current literature[27] this peculiarity of the conformal-invariant version of Einstein's dynamics has been overlooked. The energy constraint converts this dilation into a time-like classical evolution parameter which scales all masses including the Planck mass. In the conformal cosmology (CC), the evolution of the value of the massless dilation field (in the homogeneous approximation) corresponds to that of the scale factor in standard cosmology (SC). Thus, the CC is a field version of the Hoyle-Narlikar cosmology[28], where the redshift reflects the change of the atomic energy levels in the evolution process of the elementary particle masses determined by the scalar dilation field[23, 28, 29]. The CC describes the evolution of the conformal time, which has a dynamics different from that of the standard Friedmann model. Behnke *et al.*[21] have also discussed as an observational argument in favour of the CC scenario that the Hubble diagram (effective magnitude-redshift-relation: $m(z)$) including the recent SCP data[26] can be described without a cosmological constant.

Most studies in cosmology involve a perfect fluid. However, observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation suggest that we should analyse dissipative effects in cosmology. Furthermore, there are several processes

which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. The model studied by Murphy[38] possessed an interesting feature in that the big bang type of singularity of infinite space-time curvature does not occur a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity[30]–[45]. This motivates us to study a cosmological viscous fluid model.

The Einstein's field equations are a coupled system of highly nonlinear differential equations and we seek physical solutions to the field equations for their applications in cosmology and astrophysics. In order to solve the field equations we normally assume a form for the matter content or that spacetime admits killing vector symmetries[46]. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman[47]. It is interesting to observe that the law yields a constant value for deceleration parameter (DP). The variation of Hubble's law as assumed is not inconsistent with observation and has the advantage of providing simple functional forms of the scale factor. In simplest case the Hubble law yields a constant value for the DP. The cosmological models with constant deceleration parameter (CDP) may be divided into two categories. The first category of models with CDP is that of models where the cosmic expansion is driven by big bang impulse; all the matter and radiation energy is proposed at the big bang epoch and the universe has started with singular origin. In the second category of models with CDP, the universe has a non-singular origin and the cosmic expansion is driven by the creation of matter particles. It is worth observing that most of the well-known models of Einstein's theory and Brans-Deke theory with curvature parameter $k = 0$, including inflationary models, are models with CDP. It also measures the deviation from linearity of growth of the scale factor. In earlier literature cosmological models with an CDP have been studied by Berman[47], Berman and Gomide[48], Johri and Desikan[49, 50], Singh and Desikan[16], Maharaj and Naidoo[51], Pradhan *et al.*[17, 40, 42] and others. This has provided us the motivation to study models with CDP.

In this paper, we have investigated bulk viscous Locally Rotationally Symmetric (LRS) Bianchi Type I cosmological models based on Lyra's geometry with a time dependent displacement field. It is remarkable to note here that the time dependent displacement vector may lead to the singularity free model[52]. We have obtained exact solutions of the field equations by assuming the deceleration parameter to be constant. The physical behaviour of these models have been discussed.

2 Field Equations

The metric for LRS Bianchi Type I spacetime is

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2), \quad (1)$$

where, A and B are functions of x and t . The energy momentum tensor in the presence of bulk stress has the form

$$T_{ij} = (\rho + \bar{p})u_i u_j - \bar{p}g_{ij}, \quad (2)$$

together with comoving coordinates $u^i u_i = 1$ where $u_i = (0, 0, 0, 1)$ and

$$\bar{p} = p + \xi u_{;i}^i \text{ and.} \quad (3)$$

Here ρ, p, \bar{p}, ξ and u are, respectively, the energy density, isotropic pressure, effective pressure, bulk viscous coefficient and four-velocity vector of the matter distribution. Hereafter, the semi-colon denotes covariant differentiation. In general, ξ is a function of time.

The field equations in normal gauge for Lyra's manifold, as obtained by Sen [2] are

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i \phi_j - \frac{3}{4}g_{ij}\phi_k \phi^k = -8\pi GT_{ij}, \quad (4)$$

where ϕ is a time-like displacement field vector defined by

$$\phi_i = (0, 0, 0, \beta(t)), \quad (5)$$

and the other symbols have their usual meaning as in Riemannian geometry. The energy momentum tensor T^{ij} is not conserved in Lyra's geometry. Here we want to mention the fact that the *ansatz* choosing the coordinate system with matter require the vector field happens to be in the required form exactly in the matter comoving coordinates. The essential difference between the cosmological theories based on Lyra geometry and the Riemannian geometry lies in the fact that the constant vector displacement field β arises naturally from the concept of gauge in Lyra geometry whereas the cosmological constant Λ was introduced in *ad hoc* fashion in the usual treatment.

The field equations (4) with the equations (2) and (5) take the form

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{B'^2}{A^2 B^2} + \frac{3}{4}\beta^2 = -\chi\bar{p}, \quad (6)$$

$$\dot{B}' - \frac{B'\dot{A}}{A} = 0, \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{B''}{A^2 B} + \frac{A'B'}{A^3 B} + \frac{3}{4}\beta^2 = -\chi\bar{p}, \quad (8)$$

$$2\frac{B''}{A^2 B} - 2\frac{A'B'}{A^3 B} + \frac{B'^2}{A^2 B^2} - 2\frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} + \frac{3}{4}\beta^2 = \chi\rho. \quad (9)$$

The energy conservation equation is

$$\chi\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + \left[\chi(\rho + \bar{p}) + \frac{3}{2}\beta^2 \right] \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right) = 0, \quad (10)$$

where $\chi = 8\pi G$. Here and in what follows, a prime and a dot indicate partial differentiation with respect to x and t respectively. We note that the coefficient of bulk viscosity ξ does not appear explicitly in the field equations above. For the specification of ξ , we assume that the fluid obeys a barotropic equation of state

$$p = \gamma\rho, \quad (11)$$

where $\gamma(0 \leq \gamma \leq 1)$ is a constant.

3 Solutions of the field equations and discussion

On integrating the equation (7), we obtain

$$A = \frac{B'}{l}, \quad (12)$$

where l is an arbitrary function of x . Using equation (12), equations (6) and (8) can be written as

$$\frac{B}{B'} \frac{d}{dx} \left(\frac{\ddot{B}}{B} \right) + \frac{\dot{B}}{B'} \frac{d}{dt} \left(\frac{B'}{B} \right) + \frac{l^2}{B^2} \left(1 - \frac{Bl'}{B'l} \right) = 0. \quad (13)$$

Since A and B are separable functions of x , so, $\frac{B'}{B}$ is a function of x . Consequently, equation (13) gives after integration

$$B = lS(t), \quad (14)$$

where $S(t)$ is an arbitrary function of t . Using the equation (14), (12) becomes

$$A = \frac{l'}{l}S. \quad (15)$$

The metric (5) then takes the form

$$ds^2 = dt^2 - S^2(t) [dX^2 + e^{2X}(dy^2 + dz^2)], \quad (16)$$

where $X = \ln l$. Equations (6) and (9) give

$$\chi\bar{p} = \frac{1}{S^2} - 2\frac{\ddot{S}}{S} - \frac{\dot{S}^2}{S^2} - \frac{3}{4}\beta^2, \quad (17)$$

$$\chi\rho = 3\frac{\dot{S}^2}{S^2} - \frac{3}{S^2} - \frac{3}{4}\beta^2. \quad (18)$$

Using the equation (11) and eliminating $\rho(t)$ from equations (17) and (18), we have

$$2\frac{\ddot{S}}{S} + (1+3\gamma)\frac{\dot{S}^2}{S^2} - (1+3\gamma)\frac{1}{S^2} + \frac{3}{4}(1-\gamma)\beta^2 = 3\chi\xi\frac{\dot{S}}{S}. \quad (19)$$

In most of the investigations involving bulk viscosity, the coefficient of bulk viscosity is assumed to be a simple power function of the energy density [35]–[37]

$$\xi(t) = \xi_0\rho^n, \quad (20)$$

where ξ_0 and n are constants. If $n = 1$, equation (20) may correspond to a radiative fluid. However, more realistic models are based on n lying in the regime $0 \leq n \leq \frac{1}{2}$.

Using equation (20), (19) can be written as

$$2\frac{\ddot{S}}{S} + (1+3\gamma)\frac{\dot{S}^2}{S} - (1+3\gamma)\frac{1}{S^2} + \frac{3}{4}(1-\gamma)\beta^2 = 3\chi\xi_0\rho^n\frac{\dot{S}}{S}. \quad (21)$$

In order to obtain the explicit dependence of $S(t)$ on time, we assume the deceleration parameter to be constant, i.e.,

$$q = -\frac{S\ddot{S}}{\dot{S}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b \text{ (constant)}, \quad (22)$$

where $H = \frac{\dot{S}}{S}$ is the Hubble parameter. It is worthwhile to note that the solutions arrived at and the problem scanned cannot be properly handled by taking the DP to be a variable one. By considering DP to be a function of time t , we observe that q is rapidly increasing which is inconsistent and hence we should restrict ourselves to CDP. The equation (22) can be rewritten as

$$\frac{\ddot{S}}{S} + b\frac{\dot{S}^2}{S^2} = 0. \quad (23)$$

Integration of equation (23) gives us the exact solution

$$\begin{aligned} S(t) &= [a(t-t_0)]^{\frac{1}{1+b}}, \quad \text{when } b \neq -1 \\ &= m_1 e^{m_2 t}, \quad \text{when } b = -1 \end{aligned} \quad (24)$$

where a, m_1 and m_2 are constants of integration and the constant t_0 allows us the freedom of choosing the initial time.

On using equation (23) in (21), we obtain

$$[3(1+\gamma) - 2(1+b)]H^2 - (1+3\gamma)\frac{1}{S^2} + \frac{3}{4}(1-\gamma)\beta^2 = 3\chi\xi_0\rho^n H. \quad (25)$$

Equation (25) with equation (18) leads to

$$\begin{aligned} [3(1+\gamma) - 2(1+b)]H^2 &- (1+3\gamma)\frac{1}{S^2} + \frac{3}{4}(1-\gamma)\beta^2 \\ &= 3\chi^{1-n}\xi_0 H \left[3H^2 - \frac{3}{S^2} - \frac{3}{4}\beta^2 \right]^n. \end{aligned} \quad (26)$$

In what follows, we will solve the equations for two particular values of n , viz., $n = 0$ and $n = 1$.

3.1 Solutions for $\xi = \xi_0$

In this case, we assume $n = 0$ in equation (20). Equations (20) and (26) become $\xi = \xi_0 = \text{constant}$ and

$$[3(1 + \gamma) - 2(1 + b)] H^2 - (1 + 3\gamma) \frac{1}{S^2} + \frac{3}{4}(1 - \gamma)\beta^2 = 3\chi\xi_0 H. \quad (27)$$

Case (i): $b \neq -1$. For singular models, $S(0) = 0$ and hence, equation (24) leads to

$$S = a t^{\frac{1}{(1+b)}}. \quad (28)$$

In the figure 1, we have shown how the scale factor evolves with time for two choices of b , $b = 1$ and $b = 2$.

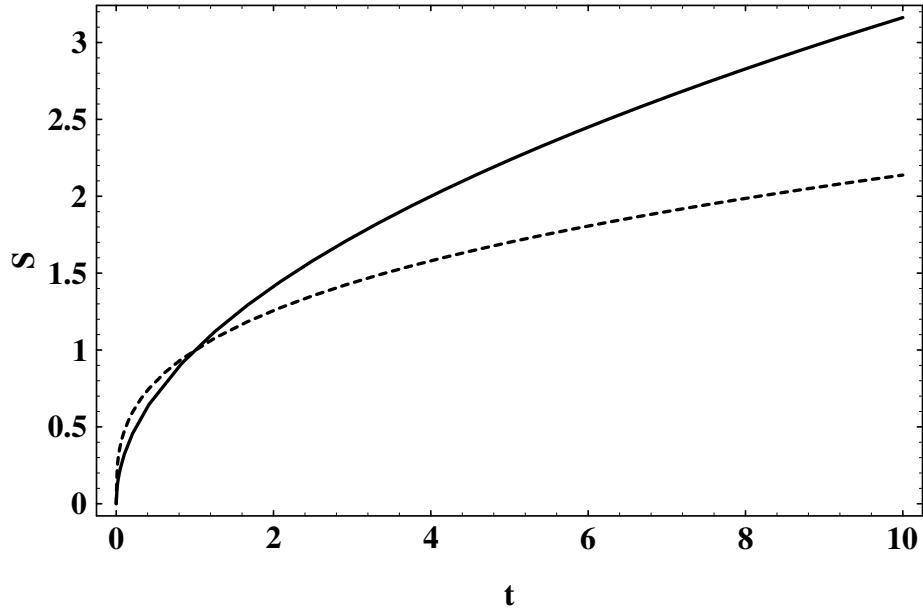


Figure 1: The behaviour of scale factor for MODEL I ($b \neq -1$) with time for $b = 1$ (solid line) and $b = 2$ (dashed line) for $a = 1$.

From equation (28), we have

$$H = \frac{1}{(1+b)t}. \quad (29)$$

The evolution of the Hubble parameter with time is shown in figure 2 for $b = 1$ and $b = 2$ for $a = 1$.

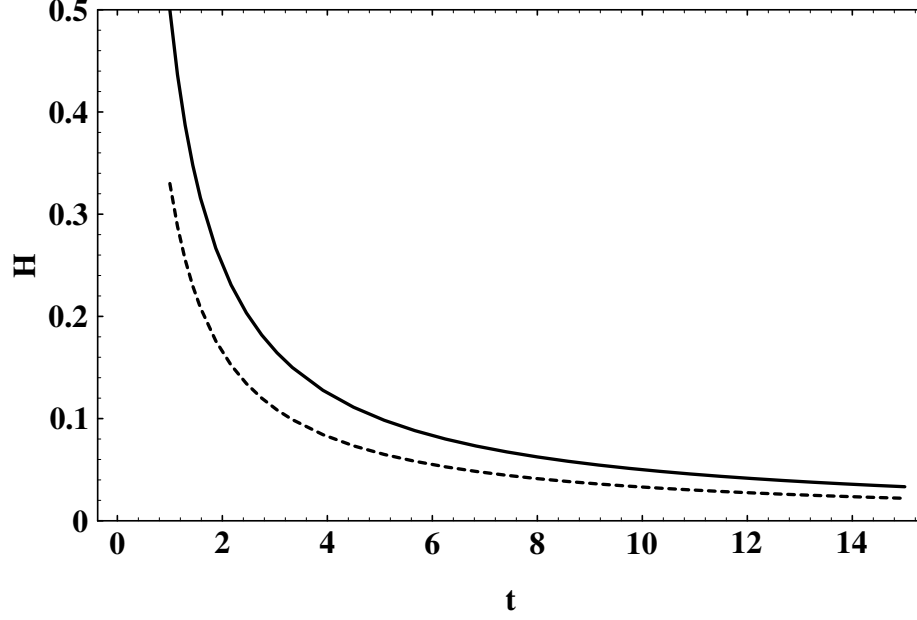


Figure 2: The behaviour of Hubble parameter for MODEL I ($b \neq -1$) with time for $b = 1$ (solid line) and $b = 2$ (dashed line).

Using the two equations (28) and (29) in the equation (27), we obtain

$$\beta^2 = \frac{4}{3} \frac{1}{(1-\gamma)(1+b)^2 t^2} \times \left[3\chi\xi_0(1+b)t - (3\gamma - 2b + 1) + \frac{(3\gamma + 1)(1+b)^2}{a^2 t^{-\frac{2b}{(1+b)}}} \right]. \quad (30)$$

Using equation (30) in (18), we get

$$\chi\rho = \frac{1}{(1-\gamma)(1+b)^2 t^2} [2(2-b) - 3\chi\xi_0(1+b)t] - \frac{4}{(1-\gamma)a^2 t^{-\frac{2}{(1+b)}}}. \quad (31)$$

The geometry of the universe, in this case, is described by the line-element

$$ds^2 = dt^2 - a^2 t^{\frac{2}{(1+b)}} [dX^2 + e^{2X}(dy^2 + dz^2)]. \quad (32)$$

In equation (30), we observe that $\beta^2 = 0$ when $t = t_c$, where the critical time t_c satisfies

$$t_c^{-\frac{2b}{(1+b)}} [(3\gamma - 2b + 1) + 3\gamma\xi_0(1+b)t_c] = \frac{(3\gamma + 1)(1+b)^2}{a^2}.$$

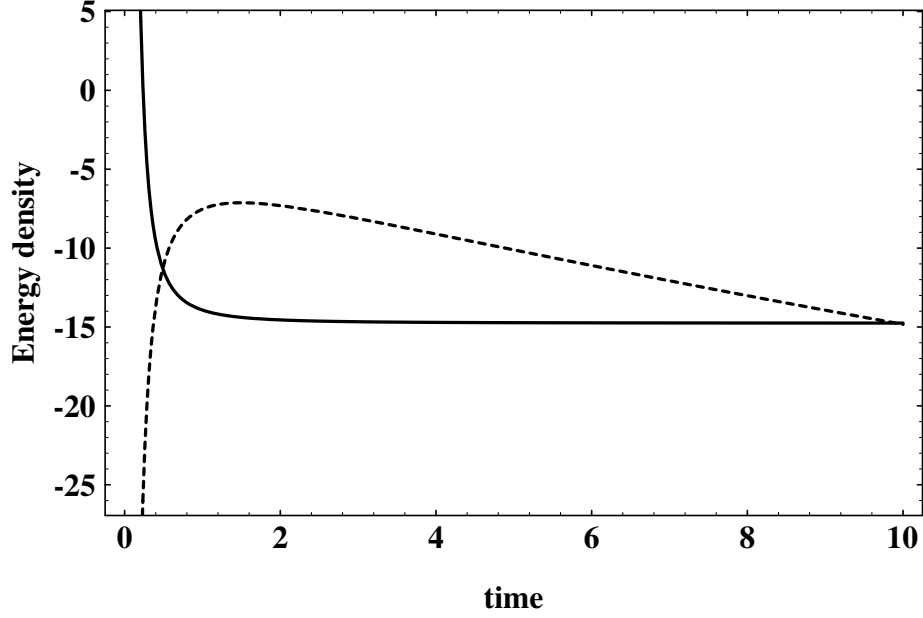


Figure 3: The plot of energy density for MODEL I ($b \neq -1$) with time for $b = 1$ (solid line) and $b = 2$ (dashed line). (Here, we have taken $\chi = 1$, $\gamma = 0.4$ and $\xi_0 = 0.5$)

It is also seen that $\beta^2 > 0$ provided $t > t_c$ and $\beta^2 < 0$ provided $t < t_c$.

From equation (31), it is also observed that $\rho > 0$ provided $t > T_c$, where T_c is the critical time given by

$$T_c^{-\frac{2b}{(1+b)}} [2(2-b) - 3\chi\xi_0(1+b)T_c] = \frac{4(1+b)^2}{a^2}.$$

The figure 3 shows the evolution of the energy density with time for two values of b , viz., $b = 1$ and $b = 2$. We see from the figure that when $t > T_c$, the energy density is positive, but when $t < T_c$, the energy density becomes negative and then evolves to a constant negative value.

Physical behaviour of the model:

In the case of a non-flat model, when $b \neq -1$, the Ricci scalar becomes

$$R = \frac{1}{a^2 t^{\frac{2}{(1+b)}}} - \frac{(1-b)t}{(1+b)}. \quad (33)$$

In equation (33), we see that when $t \rightarrow 0$; (i) $R \rightarrow \infty$ if $b = 0$, and (ii) $R \rightarrow \infty$ if $b \geq 1$. The equation (33) also suggests that when $t \rightarrow \infty$, $R \rightarrow 0$ if $b \geq 0$. The expansion and shear scalars are given by

$$\theta = \frac{3}{(1+b)t}, \quad \sigma = 0, \quad (34)$$

respectively.

The model has a singularity at $t = 0$. As $t \rightarrow \infty$, the expansion ceases. In this model, $\frac{\sigma}{\theta} = 0$, which confirms the isotropic nature of the spacetime which we have obtained in (32).

Case (ii): $b = -1$. In this case, we obtain from the equation (22)

$$\dot{H} = 0 \Rightarrow H = H_0 = \text{constant}, \quad (35)$$

and scale factor is given by

$$S = m_1 e^{m_2 t}.$$

The evolution of the scale factor with time in this case for $m_1 = 1$ and $m_2 = 2, 3$ is shown in figure 4.

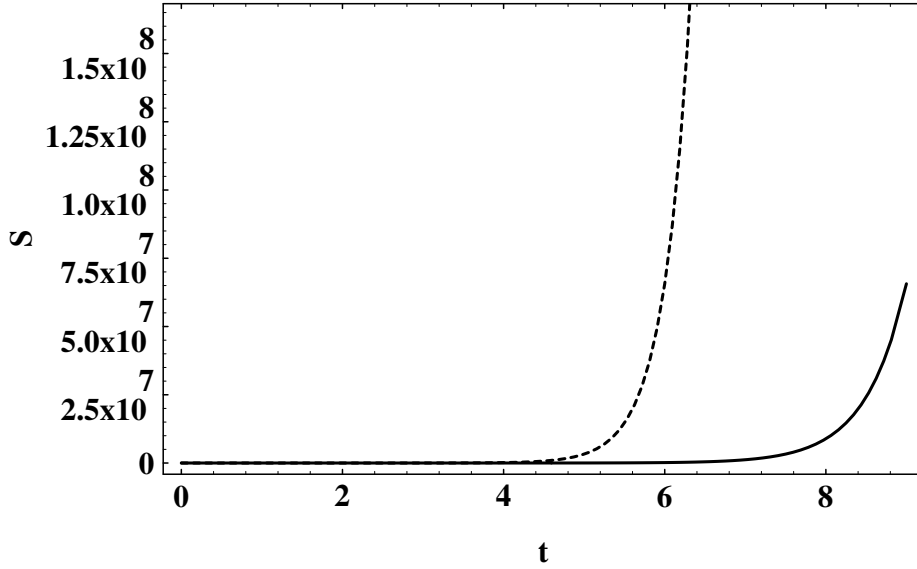


Figure 4: The behaviour of the scale factor for MODEL I ($b = -1$) with time for $m_2 = 2$ (solid line) and $m_2 = 3$ (dashed line).

On using equation (35), equations (27) and (18) reduce to

$$\beta^2 = \frac{4}{3(1-\gamma)} \left[3\chi\xi_0 H_0 - 3(1+\gamma)H_0^2 + \frac{(1+3\gamma)}{m_1^2} e^{-2m_2 t} \right], \quad (36)$$

$$\chi\rho = \frac{1}{(1-\gamma)} \left[6H_0^2 - 3\chi\xi_0 H_0 - \frac{4}{m_1^2} e^{-2m_2 t} \right]. \quad (37)$$

The geometry of the Universe, in this case, is described by the line element

$$ds^2 = dt^2 - a^2 e^{2H_0 t} [dX^2 + e^{2X} (dy^2 + dz^2)]. \quad (38)$$

From equation (36), we see that

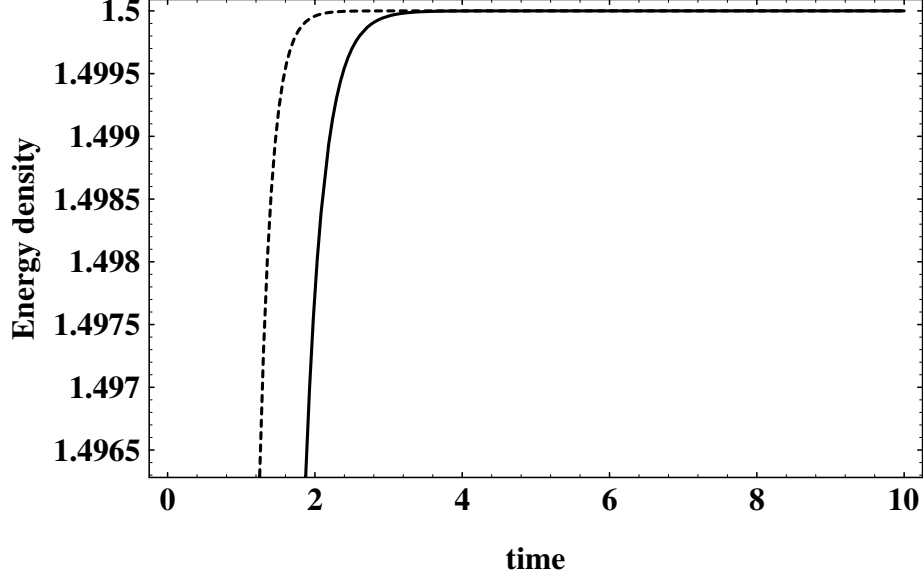


Figure 5: The behaviour of the energy density for MODEL I ($b = -1$) with time for $m_2 = 2$ (solid line) and $m_2 = 3$ (dashed line). (Here, we have taken $\chi = 1$, $m_1 = 1$, $\gamma = 0.4$, $\xi_0 = 0.4$ and $H_0 = 0.5$)

1. $\beta^2 > 0$ provided $\xi_0 < \frac{2H_0}{\chi}$ for all $t > T$,
2. $\beta^2 > 0$ provided $\xi_0 > \frac{2H_0}{\chi}$ for all $t < T$,
3. $\beta^2 < 0$ provided $\xi_0 > \frac{2H_0}{\chi}$ for all $t > T$,
4. $\beta^2 < 0$ provided $\xi_0 < \frac{2H_0}{\chi}$ for all $t < T$,
5. $\beta^2 = 0$ provided $t = T$,

where

$$T = \frac{1}{2m_2} \ln \left[\frac{(1 + 3\gamma)}{m_1^2 \{3(1 + \gamma)H_0^2 - 3\chi\xi_0 H_0\}} \right].$$

From equation (37), we observe that

- (i) $\rho > 0$, if $\xi_0 < \frac{2H_0}{\chi}$ for all $t > T_1$,
- (ii) $\rho > 0$, if $\xi_0 > \frac{2H_0}{\chi}$ for all $0 < t < T_1$,

where,

$$T_1 = \frac{1}{2m_2} \ln \left[\frac{4}{m_1^2 \{6H_0^2 - 3\chi\xi_0 H_0\}} \right].$$

In the figure 5, we have shown the evolution of the energy density with time for two values of m_2 , viz., $m_2 = 2$ and $m_2 = 3$. We see from the figure that the energy density increases with time and then approaches a constant positive value.

Physical behaviour of the model:

The Ricci scalar R is given by

$$R = 2H_0^2 - \frac{1}{a^2 e^{2H_0 t}}. \quad (39)$$

We clearly observe from equation (39) that (i) when $t \rightarrow 0, R \rightarrow (2H_0^2 - \frac{1}{a^2})$, and (ii) when $t \rightarrow \infty, R \rightarrow 2H_0^2$. The expansion and shear scalars are

$$\theta = 3H_0, \quad \sigma = 0. \quad (40)$$

This model represents an uniform expansion as can be seen from equation (40). The flow of the fluid is geodesic as the acceleration vector $f_i = (0, 0, 0, 0)$.

3.2 Solutions for $\xi = \xi_0 \rho$

In this case, we choose $n = 1$ and hence, equations (20) and (26) now reduce to

$$\xi = \xi_0 \rho, \quad (41)$$

$$\begin{aligned} [3(1 + \gamma) - 2(1 + b)] H^2 - (1 + 3\gamma) \frac{1}{S^2} + \frac{3}{4}(1 - \gamma)\beta^2 = \\ 3\xi_0 H \left(3H^2 - \frac{3}{S^2} - \frac{3}{4}\beta^2 \right). \end{aligned} \quad (42)$$

respectively.

Equation (42) can be written as

$$\beta^2 = \frac{4}{3} \frac{9\xi_0 H^3 - (3\gamma - 2b + 1)H^2 + (1 + 3\gamma - 9\xi_0 H)S^{-2}}{(1 - \gamma + 3\xi_0 H)}. \quad (43)$$

Case (i): $b \neq -1$.

Here we are considering a singular model. Hence, using equations (28) and (29) in equations (43) and (18), we obtain

$$\begin{aligned} \beta^2 = \frac{4}{3a^2(1+b)^2 t^2 [(1-\gamma)(1+b)t + 3\xi_0]} \times \\ [9a^2 \xi_0 - 9\xi_0(1+b)^2 t^{\frac{2b}{1+b}} - (3\gamma - 2b + 1)(1+b)a^2 t + (1 + 3\gamma)(1+b)^3 t^{\frac{3b+1}{1+b}}] \end{aligned} \quad (44)$$

$$\chi \rho = \frac{2}{a^2(1+b)t} \frac{[(2-b)a^2 - 2(1+b)^2 t^{\frac{2b}{1+b}}]}{[(1-\gamma)(1+b)t + 3\xi_0]}. \quad (45)$$

Therefore, in this case, from equation (44), we see that

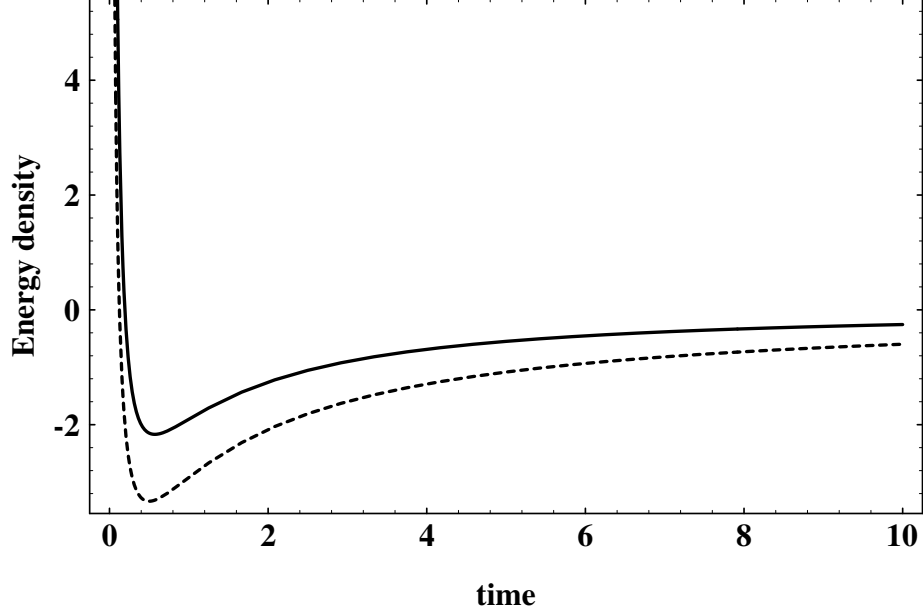


Figure 6: The behaviour of the energy density for MODEL II ($b \neq -1$) with time for $b = 0.5$ (solid line) and $b = 1$ (dashed line). (Here, we have taken $\chi = 1$, $a = 1$, $\gamma = 0.4$ and $\xi_0 = 0.4$)

1. $\beta^2 = 0$ if $t = t_c$,
2. $\beta^2 > 0$ if $t = t_c$,
3. $\beta^2 < 0$ if $t < t_c$,

where, the critical time t_c satisfies the relation

$$(1+b)^2 t_c^{\frac{2b}{(1+b)}} [9\xi_0 - (1+3\gamma)(1+b)t_c] = 9a^2\xi_0 - (3\gamma - 2b + 1)(1+b)a^2t_c.$$

It can also be seen from equation (45) that $\rho > 0$ if $b < 2$ with $t > 0$. The behaviour of the energy density with time is shown in the figure 6 for $b = 0.5$ and $b = 1$. We see from the figure that the energy density approaches a small positive value after initially becoming negative for a short period of time. The physical properties of this model are similar to those discussed for model I.

Case (ii): $b = -1$.

As in the previous model, we now have

$$H = H_0 = \text{constant}. \quad (46)$$

Equations (43) and (18) become

$$\beta^2 = \frac{4}{3} \frac{[9\xi_0 m_1^2 H_0^3 - 3(1+\gamma)H_0^2 m_1^2 + (1+3\gamma - 9\xi_0 H_0)e^{-2m_2 t}]}{[1 - \gamma + 3\xi_0 H_0]m_1^2}, \quad (47)$$

$$\chi\rho = \frac{2}{(1 - \gamma + 3\xi_0 H_0)m_1^2} [(3\gamma - 1)e^{-2m_2 t} - 3\gamma m_1^2 H_0^2]. \quad (48)$$

In equation (47), we observe that

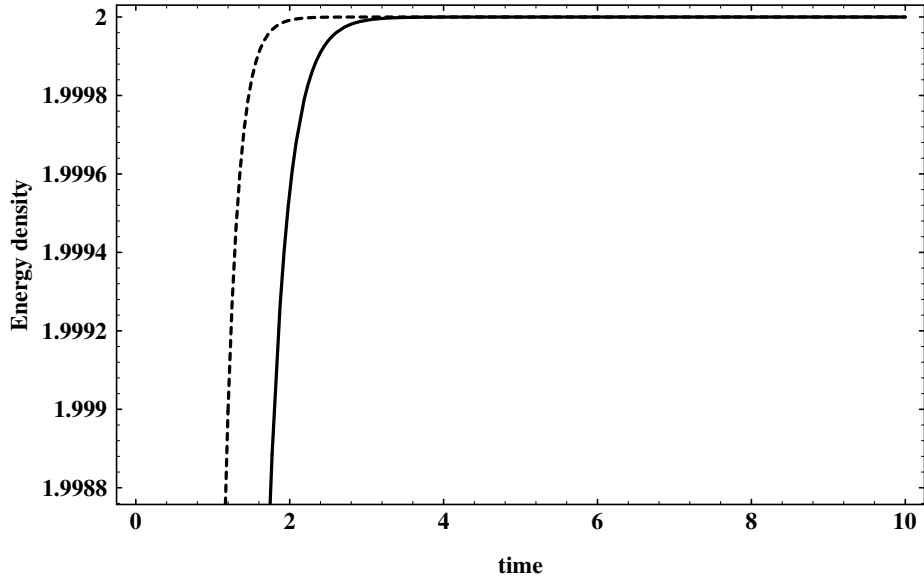


Figure 7: The behaviour of the energy density for MODEL II ($b = -1$) with time for $m_2 = 2$ (solid line) and $m_2 = 3$ (dashed line). (Here, we have taken $m_1 = 1$, $\gamma = 0.4$, $\xi_0 = -0.6$, and $H_0 = 0.5$)

1. $\beta^2 > 0$ if $\xi_0 < \frac{(1+\gamma)}{3H_0}$ for all $t < T_2$,
2. $\beta^2 > 0$ if $\xi_0 > \frac{(1+\gamma)}{3H_0}$ for all $t > T_2$,
3. $\beta^2 < 0$ if $\xi_0 < \frac{(1+\gamma)}{3H_0}$ for all $t > T_2$,
4. $\beta^2 < 0$ if $\xi_0 > \frac{(1+\gamma)}{3H_0}$ for all $t < T_2$,
5. $\beta^2 = 0$ if $\xi_0 \neq \frac{(1+\gamma)}{3H_0}$ for all $t = T_2$,

where

$$T_2 = \frac{1}{2m_2} \ln \left[\frac{(1 + 3\gamma - 9\xi_0 H_0)}{3m_1^2 H_0^2 (1 + \gamma - 3\xi_0 H_0)} \right].$$

From the equation (48), we see that

$$\begin{aligned} (i) \quad & \rho > 0, \quad \text{provided } \xi > \frac{(\gamma - 1)}{3H_0} \text{ for all } t > T_3, \\ (ii) \quad & \rho > 0, \quad \text{provided } \xi < \frac{(\gamma - 1)}{3H_0} \text{ for all } t < T_3, \end{aligned}$$

where

$$T_3 = \frac{1}{2m_2} \ln \left[\frac{3\gamma - 1}{3\gamma m_1^2 H_0^2} \right].$$

The behaviour of the energy density with time is shown in figure 7. We see that the energy density increases with time and approaches a constant positive value. The physical properties of this model is similar to those of model I.

4 Discussion and Conclusion

In this paper, we have investigated LRS Bianchi type I models with a bulk viscous fluid and obtained exact solutions for a constant deceleration parameter. We have assumed the coefficient of bulk viscosity to be of the form $\xi(t) = \xi_0 \rho^n$; where ρ is the energy density and n is the power index. The behaviour of the displacement field β and the energy density have been examined for values of $n = 0$ and $n = 1$ for both (i) power-law and (ii) exponential expansion of a non-flat universe. The model discussed here is isotropic and homogeneous and in view of the assumption of isotropy, the shear viscosity cannot exist. The effect of bulk viscosity is to introduce a change in the perfect fluid model.

Recently there is an upsurge of interest in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary cosmology [53]–[55]. Therefore the study of cosmological models in Lyra geometry may be relevant for inflationary models. Further the space dependence of the displacement field β is important for inhomogeneous models for the early stage of the evolution of the universe. Besides, the implication of Lyra's geometry for astrophysical interesting bodies is still an open question. The problem of equations of motion and gravitational radiation need investigation. Finally in spite of very good possibility for Lyra's geometry to provide a theoretical foundation for Relativistic Gravitation, Astrophysics and Cosmology, the experimental point is yet to be undertaken. But still the theory needs a fair trial.

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